Name: Solutions

Date: 4/10/2025

Math 130
Exam 3

Please show ALL your work on the problems below. No more than 1 point will be given to problems if *you only provide the correct answer and insufficient work.*

1. (10 points) State the Central Limit Theorem (for $\overline{X}$)

If X is any random variable and $n \geq 30$, then $\overline{X}$ has a normal distribution.
If X has a normal distribution, then $\overline{X}$ has a normal distribution no matter what n is:

$$\mu_{\overline{X}} = \mu_X \qquad \sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}}$$

2. (8, 15 points) The lengths of TV commercials have a normal distribution with a mean of 42.7 seconds and a standard deviation of 4.3 seconds.

a) What is the probability that a randomly selected TV commercial lasts longer than 50 seconds?

X = The length of a randomly selected TV commercial

X is normal (given)

$\mu_X = 42.7$ sec.

$\sigma_X = 4.3$ sec.

$$P(X > 50) \overset{\text{z-trans}}{=} P\left(\frac{X-\mu_X}{\sigma_X} > \frac{50-\mu_X}{\sigma_X}\right)$$
$$= P\left(Z > \frac{50-42.7}{4.3}\right) = P(Z > 1.70)$$
$$= 1 - P(Z < 1.70) = 1 - 0.9554$$
$$= \boxed{0.0446}$$

b) After watching TV for an hour, you find that there were 26 commercials. What is the probability that the average length of these commercials is between 41 and 42 seconds?

$\overline{X}$ = The average length of the 26 commercials

$\overline{X}$ is normal (by CLT bec. X is normal)

$\mu_{\overline{X}} = \mu_X = 42.7$ sec.

$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{4.3}{\sqrt{26}}$

$$P(41 < \overline{X} < 42) \overset{\text{z-trans}}{=} P\left(\frac{41-\mu_{\overline{X}}}{\sigma_{\overline{X}}} < \frac{\overline{X}-\mu_{\overline{X}}}{\sigma_{\overline{X}}} < \frac{42-\mu_{\overline{X}}}{\sigma_{\overline{X}}}\right)$$
$$= P\left(\frac{41-42.7}{\frac{4.3}{\sqrt{26}}} < Z < \frac{42-42.7}{\frac{4.3}{\sqrt{26}}}\right)$$
$$= P(-2.02 < Z < -0.83)$$
$$= P(Z < -0.83) - P(Z < -2.02)$$
$$= 0.2033 - 0.0217 = \boxed{0.1816}$$

3. (15 points) 8% of all males in the world are colorblind. In a group of 145 males, what is the probability that more than 6% of them are colorblind?

X = The yes or no answer when a randomly selected male is asked if he is colorblind.

Dist: unknown

$p = 0.08$

$q = 0.92$

$\hat{p}$ = The percentage in a randomly selected group of 145 males that are colorblind

Condition: $npq \geq 10$?

$(145)(0.08)(0.92) = 10.672 \geq 10$ ✓

Dist: Normal (by CLT bec $npq \geq 10$)

$\mu_{\hat{p}} = p = 0.08$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.08)(0.92)}{145}}$

$P(\hat{p} > 0.06) \overset{\text{z-trans}}{=} P\left(\frac{\hat{p}-\mu_{\hat{p}}}{\sigma_{\hat{p}}} > \frac{0.06-\mu_{\hat{p}}}{\sigma_{\hat{p}}}\right)$

$= P\left(Z > \frac{0.06-0.08}{\sqrt{\frac{(0.08)(0.92)}{145}}}\right)$

$= P(Z > -0.89)$

$= 1 - P(Z < -0.89)$

$= 1 - 0.1867$

$= \boxed{0.8133}$

4. (15 points) In order to determine the percentage of all adults in North America that are millionaires, 700 adults from North America were randomly selected and their net worth was calculated. Of the 700 people selected, it was determined that 44 of them are millionaires. Find a 93% confidence interval for the percentage of all adults in North America that are millionaires.

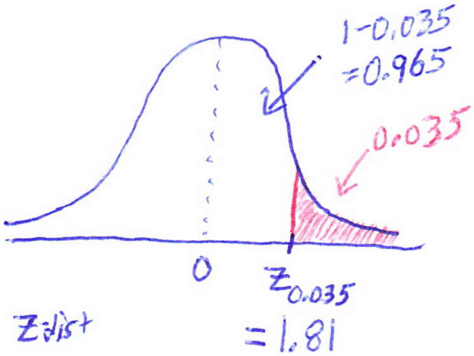
$\hat{p} = \frac{44}{700} = 0.0629 = 6.29\%$

$z_{\alpha/2} = ?$

$\alpha = 1 - \text{conf. level} = 1 - 0.93 = 0.07$

$\alpha/2 = \frac{0.07}{2} = 0.035$

$z_{\alpha/2} = z_{0.035} = 1.81$



$E = ?$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = (1.81)\sqrt{\frac{(.0629)(.9371)}{700}}$

$= 0.0166 = 1.66\%$

Interval

$\hat{p} - E < p < \hat{p} + E$

$6.29\% - 1.66\% < p < 6.29\% + 1.66\%$

$\boxed{4.63\% < p < 7.95\%}$

5. (3, 3, 3, 5, 15, 8 points) In order to figure out the average weight of all current NBA players, 19 players were randomly selected and weighed. After the data was collected, it was determined that these 19 players had an average weight of 214.6 pounds with a standard deviation of 8.2 pounds.

a) What is the population?

All current NBA players

b) What is the sample?

The 19 randomly selected NBA players

c) What is the population parameter? (symbol and in words)

μ = The average weight of all NBA players.

d) What is the best point estimate for the population parameter we are trying to estimate?

$\mu \approx \bar{x} = 214.6$ pounds

e) Find a 90% confidence interval for the average weight of all current NBA players

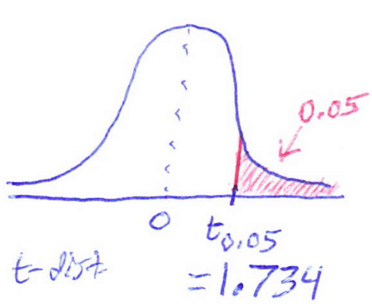
$t_{\alpha/2} = ?$

$\alpha = 1 - \text{conf. level} = 1 - 0.90 = 0.10$

$\alpha/2 = 0.10/2 = 0.05$

$df = n-1 = 19-1 = 18$

$t_{\alpha/2} = t_{0.05} = 1.734$



$E = ?$

$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

$= (1.734)\frac{8.2}{\sqrt{19}}$

$= 3.26$ pounds

Interval

$\bar{x} - E < \mu < \bar{x} + E$

$214.6 - 3.26 < \mu < 214.6 + 3.26$

$\boxed{211.34 \text{ pounds} < \mu < 217.86 \text{ pounds}}$

f) What does the 90% in a 90% confidence interval mean?

If you take many samples and use them to build many 90 % confidence intervals, about 90 % of the intervals will contain the correct answer for μ and about 10 % of the intervals will not contain the correct answer for μ.

6. (6, 15 points) The following data represent the battery life, in hours, for a random sample of 10 fully charged brand new Iphone 12 batteries.

7.3	10.2	12.9	10.1	13.5
6.6	7.2	8.0	8.2	7.4

a) Find the best point estimate for the standard deviation of the battery life of all new fully charged Iphone 12's.

$\Sigma x^2 = (7.3)^2 + (10.2)^2 + \cdots + (7.4)^2 = 889.4$

$\Sigma x = 7.3 + 10.2 + \cdots + 7.4 = 91.4$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{889.4 - \frac{(91.4)^2}{10}}{10-1}} = 2.45$$

$\boxed{\sigma \approx s = 2.45 \text{ hrs}}$

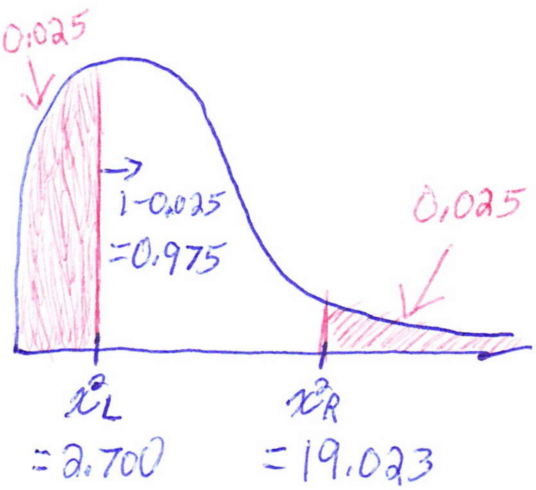
b) Construct a 95% confidence interval for the standard deviation of the battery life of all new fully charged Iphone 12's.

$\chi^2_L = ?$ $\chi^2_R = ?$

$\alpha = 1 - \text{conf. level} = 1 - 0.95 = 0.05$

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

$df = n - 1 = 10 - 1 = 9$



χ^2-dist.

Interval

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(10-1)(2.45)^2}{19.023}} < \sigma < \sqrt{\frac{(10-1)(2.45)^2}{2.700}}$$

$$\boxed{1.69 \text{ hrs} < \sigma < 4.47 \text{ hrs}}$$

Some formulas you may need:

$$Z = \frac{X - \mu}{\sigma} \qquad \mu_{\bar{X}} = \mu_X \qquad \sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} \qquad \mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad E = t_{\alpha/2}\frac{s}{\sqrt{n}} \qquad df = n - 1$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}} \qquad \frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

$$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$